Extended Abstracts

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Matlis duality, inverse systems and classification of Artin algebras

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Amb l'objectiu d'estudiar la classificació d'algunes famílies d'àlgebres artinianes, estudiarem alguns resultats importants sobre mòduls injectius i la dualitat de Matlis. Més específicament, estudiarem la correspondència de Macaulay, la dualitat de Matlis quan $R = k[[x_1, \dots, x_n]]$ amb l'ideal maximal $m = (x_1, \dots, x_n)$. Utilitzant aquesta, podem introduir-nos en les funcions de Hilbert, essencials en la classificació d'àlgebres artinianes. Tots aquests resultats han anat acompanyats de càlculs i exemples fets amb SINGULAR [3] i la llibreria INVERSE-SYST.LIB [4] del Dr. Joan Elias.

Keywords: *Matlis duality, inverse system, Artin algebra,* SINGULAR.

Abstract

Classifying Artin algebras could be a huge challenge in some cases. Luckily, we could use some theorems and results which make that process easier. We are talking about Matlis duality which will be defined later. Should one want to understand such duality, injective modules and other results should be studied previously. After that, Macaulay's correspondence, a particular case of Matlis duality, will be defined too.

As it has been said before, injective modules are the starting point. Let R be a commutative ring and E an injective R-module. By definition, we say that E is injective if and only if, for all injective morphism $i\colon A \to B$ and for all morphism $f\colon A \to E$, where A and B are R-modules, a morphism $g\colon B \to E$ exists such that all commute. Now, the existence of injective hulls of modules could be proven, essential for deducing the existence of minimal injective resolutions of R-modules.

In addition, when R is a Noetherian ring, we can define what a Bass number is and the relation between those and the minimal injective resolution of a finite R-module.

Now, it is time to define the Matlis duality. It ensures an isomorphism between Artin and Noetherian modules. Given A an R-module, let $(R, \mathfrak{a}, \mathbf{k})$ Noetherian local ring, its dual will be $A^\vee = \mathrm{Hom}_R(A, E)$, E an injective hull of $\mathbf{k}$, by Matlis duality. It was written and proved by Eben Matlis in 1958.

It is important to shed light on the particular case of Matlis duality: Macaulay's correspondence. In that scenario,

$$R = \mathbf{k}[[x_1, \dots, x_n]],$$

the ring of formal series, with maximal ideal $\mathfrak{m} = (x_1, \dots, x_n)$. Studying this correspondence will be a useful tool. Not only will it be relevant when approaching the relation between Artin rings and Gorenstein rings, but also studying level rings. Level rings are the starting point when defining Irrabino's Q-decomposition of the associated graded ring of an Artinian s-level local k-algebra.

Macaulay's correspondence could be used to study how to achieve isomorphism classes of local algebras. Furthermore, an important result is reached, an isomorphism between two Artinian s-level algebras is defined by a matrix using Macaulay's inverse system.

Through all this project, some computations have been done with SINGULAR and INVERSE-SYST.LIB by J. Elias. As one of the main and last results, one can prove the existence of an isomorphism between some models for A and its inverse system when A is an Artin Gorenstein local $\mathbf{k}$-algebra with Hilbert function $\mathrm{HF}_A = \{1, 3, 3, 1\}$. Had it not been for SINGULAR, this proof would have been large and tedious.

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